MMJ12503 – Computer Programming

Lab module

Array

1. Problem statement

The analysis of an electrical circuit frequently involves finding the solution to a set of simultaneous equations. These equations are often derived using either current equations that describe the current entering and leaving a node or using voltage equations that describe the voltages around mesh loops in the circuit. Consider the circuit as shown in Figure 1.

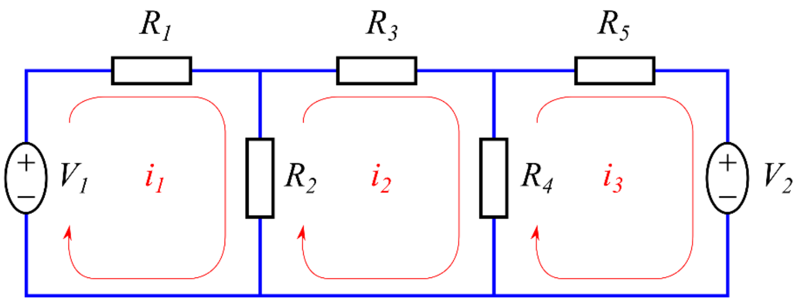

Figure 1 : Circuit with two voltage sources

The voltages around the three loops can be described with the following equations:

$$
\begin{aligned}
-V_1 &+ R_1 i_1 + R_2(i_1 - i_2) = 0 \\
R_2(i_2 - i_1) &+ R_3 i_2 + R_4(i_2 - i_3) = 0 \\
R_4(i_3 - i_2) &+ R_5 i_3 + V_2 = 0
\end{aligned}
$$

If the values of the resistors (R_1, R_2, R_3, R_4 and R_5) and the voltage sources (V_1 and V_2) are known, then the mesh currents (i_1, i_2 and i_3) are unknown. The system equations can be re-arrange as follow: -

$$
\begin{aligned}
(R_1 + R_2)i_1 - R_2 i_2 + (0)i_3 &= V_1 \\
-R_2 i_1 + (R_2 + R_3 + R_4)i_2 - R_4 i_3 &= 0 \\
(0)i_1 - R_4 i_2 + (R_4 + R_5)i_3 &= -V_2
\end{aligned}
$$

Write a C program to compute the three mesh currents in the circuit shown in Figure 1.

2. Input/output Description

Figure 2 shows the inputs to the program are the resistor values and the voltage values. The output is the current values of the three mesh currents.

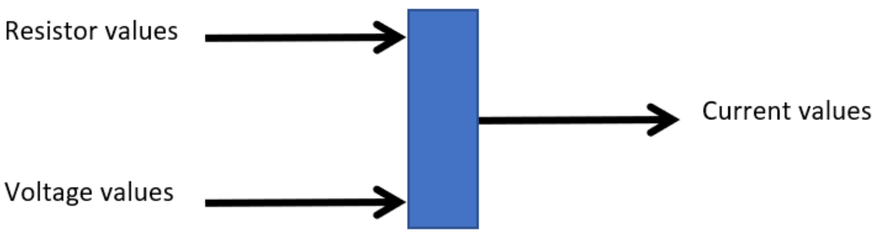

Figure 2 : Input and output of program

3. Hand example

By using the resistor values an the voltage values, a system of three equations can be defined using this rearranged set of equations from the problem definition:

$$\begin{aligned}(R_1+R_2)i_1 - \quad R_2 i_2 \quad + \quad (0)i_3 &= V_1\\ -R_2 i_1 \quad + (R_2+R_3+R_4)i_2 - \quad R_4 i_3 &= 0\\ (0)i_1 \quad - \quad R_4 i_2 \quad + (R_4+R_5)i_3 &= -V_2\end{aligned}$$

Suppose that each of the resistor values is 1 ohm, and assume that both of the voltage sources are 5 volts. Then, the corresponding set of equations is the following:

$$\begin{aligned}2i_1 - i_2 + (0)i_3 &= 5\\ -i_1 + 3i_2 - i_3 &= 0\\ (0)i_1 - i_2 + 2i_3 &= -5\end{aligned}$$

Once the system of equations is determined, the solution can be determined from matrix as follow:-

$$\begin{bmatrix}2 & -1 & 0\\ -1 & 3 & -1\\ 0 & -1 & 2\end{bmatrix}\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix} = \begin{bmatrix}5\\ 0\\ -5\end{bmatrix}$$

The Gauss elimination is consists of two main process, which are elimination and back substitution. The first step of Gauss elimination is elimination, which adding a scaled form of the first equation to each of the other equations.

$$\begin{aligned}2i_1 - i_2 + (0)i_3 &= 5 \quad \text{(1st equation)}\\ -i_1 + 3i_2 - i_3 &= 0 \quad \text{(2nd equation)}\\ (0)i_1 - i_2 + 2i_3 &= -5 \quad \text{(3rd equation)}\end{aligned}$$

The term involving the first variable (i_1 in this example), in the second equation is -i_1.

$$\begin{aligned}-i_1 + 3i_2 - i_3 &= 0 \quad \text{(2nd equation)}\\ i_1 - (1/2)i_2 + (0)i_3 &= 5/2 \quad \text{(1st equation } \times 1/2)\\ \hline (0)i_1 + (5/2)i_2 - i_3 &= 5/2 \quad \text{(sum)}\end{aligned}$$

The modified set of equations is then

$$\begin{aligned}2i_1 - i_2 + (0)i_3 &= 5 \quad \text{(1st equation)}\\ (0)i_1 + (5/2)i_2 - i_3 &= 5/2 \quad \text{(2nd equation)}\\ (0)i_1 - i_2 + 2i_3 &= -5 \quad \text{(3rd equation)}\end{aligned}$$

Now eliminate the first variable from the third equation, using a similar process:

$$\begin{aligned}(0)i_1 - i_2 + 2i_3 &= -5 \quad \text{(3rd equation)}\\ (0)i_1 + (0)i_2 - (0)i_3 &= 0 \quad \text{(1st equation } \times 0)\\ \hline (0)i_1 - i_2 + 2i_3 &= -5 \quad \text{(substract)}\end{aligned}$$

The modified set of equations is then

$$\begin{aligned}2i_1 - i_2 + (0)i_3 &= 5 \quad \text{(1st equation)}\\ (0)i_1 + (5/2)i_2 - i_3 &= 5/2 \quad \text{(2nd equation)}\\ (0)i_1 - i_2 + 2i_3 &= -5 \quad \text{(3rd equation)}\end{aligned}$$

The first variable in all equations except for the first equation are eliminated.

The next step is to eliminate the second variable in all equations except for the first and second equations. Thus, the equations is added to a scaled form of the second equation:

$$\begin{aligned}(0)i_1 - i_2 + 2i_3 &= -5 \quad \text{(3rd equation)}\\ (0)i_1 + i_2 - (2/5)i_3 &= 1 \quad \text{(2nd equation } \times 2/5)\\ \hline (0)i_1 - (0)i_2 + (8/5)i_3 &= -4 \quad \text{(sum)}\end{aligned}$$

The modified set of equations is then

$$\begin{aligned}2i_1 - i_2 + (0)i_3 &= 5 \quad \text{(1st equation)}\\ (0)i_1 + (5/2)i_2 - i_3 &= 5/2 \quad \text{(2nd equation)}\\ (0)i_1 - (0)i_2 + (8/5)i_3 &= -4 \quad \text{(3rd equation)}\end{aligned}$$

Because there are no equation following the third equation, this part of the algorithm is completed. The second step of Gauss elimination is back substitution to determine the solution to the equations. The last equation has only one variable, wo we can multiply the equation by a scale factor chosen to make the variable's coefficient equal to 1. In this example, the equation 3 is multiply with $5/8$, giving

$$(0)i_1 - (0)i_2 + i_3 = -5/2$$

The value of i_3 is substituting in the next-to-last equation, giving

$$(0)i_1 + (5/2)i_2 - (-5/2) = 5/2$$

Reducing the equation so that all constant terms are on the right side, we have

$$(0)i_1 + (5/2)i_2 = 0$$

This equation has only one variable, so we multiply it by a scale factor chosen to make the new coefficient equal to 1:

$$(0)i_1 + i_2 = 0$$

We back up to the next equation, which is the last equation in this example:

$$2i_1 - i_2 + (0)i_3 = 5$$

Substituting the values already determined, we have

$$2i_1 - (0) + (0)(-5/2) = 5$$

or

$$2i_1 = 5$$

Thus, the value of i_1 is $5/2$. For this set of equations, the solution is $i_1 = 2.5$, $i_2 = 0$ and $i_3 = -2.5$.

4. Algorithm development

Decomposition outline of the main program:-

i. Read the resistor values and the voltage values.
ii. Specify the coefficients for the system of equations.
iii. Perform Gauss elimination to determine currents.
iv. Print currents.

To keep main program short and readable, functions are used for both elimination and back substitution

Decomposition outline of the function eliminate(a,n,index):-

i. Set row to index +1.
ii. While row ≤ n -1.
   a. Set scale_factor to $\frac{-a[\text{row}][\text{index}]}{a[\text{index}][\text{index}]}$.
   b. Set $a[\text{row}][\text{index}]$ to zero.
   c. Set col to index +1.
   d. While col ≤ n,
      i. Add $a[\text{index}][\text{col}] \cdot \text{scale\_factor}$ to $a[\text{row}][\text{col}]$
      ii. Increment col by 1.
   e. Increment row by 1.

Decomposition outline of the function back_substitution(a,n,soln):-

i. Set $soln[n-1]$ to $\frac{-a[n-1][n]}{a[n-1][n-1]}$.
ii. Set row to n -2.

iii. While row ≥ 0
   a. Set col to n -1.
   b. While col ≥ row +1
      i. Substract $soln[\mathrm{col}] \cdot a[\mathrm{row}][\mathrm{col}]$ from $a[\mathrm{row}][n]$
   c. Substract 1 from col

iv. Set $soln[\mathrm{col}]$ to $\frac{-a[\mathrm{row}][n]}{a[\mathrm{row}][\mathrm{row}]}$

v. Subtract 1 from row

5. Write a C program

Please convert the decomposition directly into C.